Lecture 14 - March 16

Model Checking

LTL Examples: Until, Weak Until, Release
Formulating Natural Language in LTL

Announcements

✓ 6:45 ? lecture video

- Mar 23 class?
- ProgTest1 result to be released by the end of Friday
- Lab3 released
- WrittenTest2 example questions to be released

PPY & D.N. eecs login.

- Review Q&A session: 7pm on Sunday, March 19?

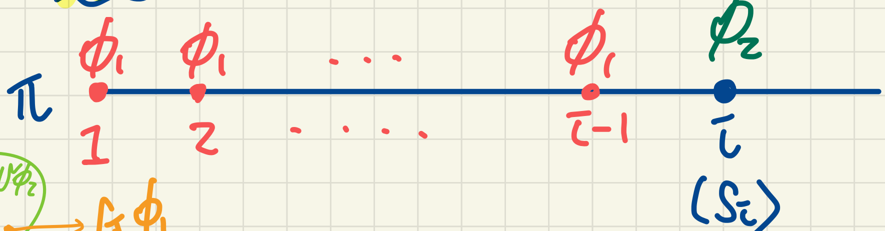
conf.

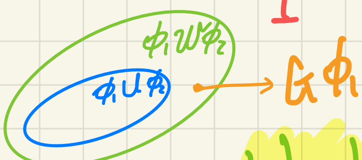
U, W, R

Until $\phi_1 \,U\, \phi_2$

$\boxed{\phi_1 \;\; \phi_1 \; \ldots}$

starting from S_i ; ϕ_1 being true or not would not impact the evaluation of the formula

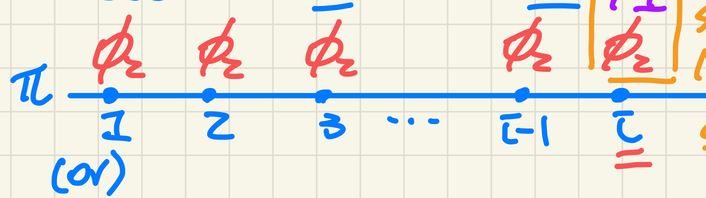


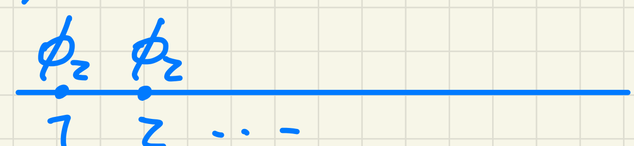


Weak Until $\phi_1 \,W\, \phi_2$

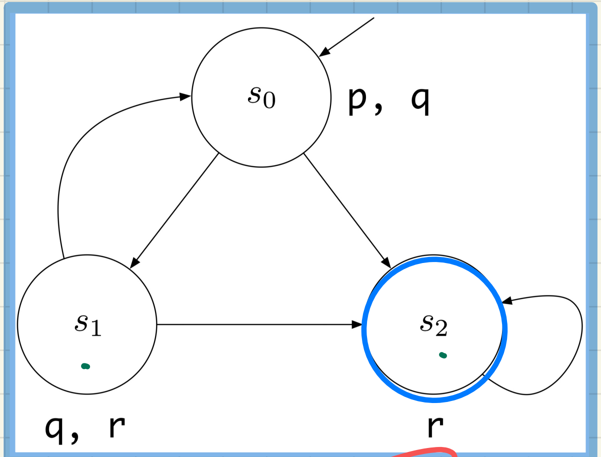
Release $\underline{\underline{\phi_1}} \,R\, \underline{\underline{\phi_2}}$

if there's no state satisfying $\phi_1 \wedge \phi_2$, then this scenario cannot satisfy.





Model Satisfaction: Exercises (7.1)



$s \models \phi \Leftrightarrow$ all π starting at s, $\pi \models \phi$

$s_0 \models p \; U \; r$ True

p: $s_0 \rightarrow s_1 \rightarrow \cdots$ (r)

$s_0 \rightarrow s_2 \rightarrow \cdots$

$(i-1)$ (i)

$s_0 \models p \; W \; r$ True

$\because \phi_1 \, U \, \phi_2 \Rightarrow \phi_1 \, W \, \phi_2$

$s_2 \models r \; U \; p$ false

$\because$ no state satisfying p

$\rightarrow s_2 \models r \; W \; p$

always true

True

* $s_2 \models p \; U \; r$ True

$s_2 \rightarrow s_2 \rightarrow \cdots$

$i = 1$

r already satisfied and $(\forall j \bullet 1 \leq j \leq i-1 \Rightarrow \pi^j \models p)$ true

$s_0 \models r \; R \; p$

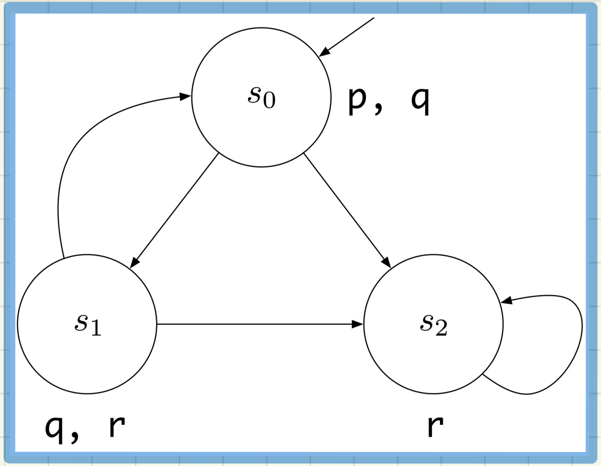
Witness $s_0 \rightarrow s_1 \rightarrow s_0 \rightarrow \cdots$

(1) no state that satisfies $p \wedge r$

(2) not the case that p is always true

Exercise: What if we change the LHS to s_2?

Model Satisfaction: Exercises (7.2)



$s \models \phi \Leftrightarrow$ all π starting at s, $\pi \models \phi$

$s_0 \models (p \lor r)\ \mathsf{U}\ (p \land r)$ false

$\because$ $p \land r$ is never satisfie

$s_0 \models (p \lor r)\ \mathsf{W}\ (p \land r)$ true

↳ We know: $(p \lor r)\ \mathcal{W}\ (p \land r)$ false

But: $(p \lor r)$ is always satisfied.

$s_0 \models (p \land r)\ \mathsf{R}\ \boxed{(p \lor r)}$

always true

$\boxed{\text{2nd case of } \mathsf{R} \text{ satisfied}}$

$s_0 \models G\,(p \lor r)$

Exercise: What if we change the LHS to s_2?

Formulating Natural Language in LTL (1)

Eg. I smoked $\mathcal{U}$
G(I was 22 ∧ ..).

Natural Language:

I had smoked until I was 22.

I was 22

s s s ... s ¬s ¬s ¬s

Atom t: I was 22

Atom s: I smoke

Q. Is s U t an appropriate formulation?

Eg.

I smoked $\mathcal{U}$

(I was 22 ∧

¬(I smoked))

s s s s

s s s

i

I was 22

$\pi^i \models t$

$\forall j \cdot 1 \le j \le i-1 \Rightarrow \pi^j \models s$

Solution

ϕ_1

I smoked $\mathcal{U}$ ϕ_2

(I was 22 ∧

G(¬ I smoked))

$$\pi \models \phi_1 \, \mathbf{U} \, \phi_2 \iff \left(\exists i \bullet i \geq 1 \land \left(\begin{array}{l} \pi^i \models \phi_2 \ldots \land G(\ldots) \\ \land \\ (\forall j \bullet 1 \leq j \leq i-1 \Rightarrow \pi^j \models \phi_1) \end{array} \right) \right)$$

Formulating Natural Language in LTL (2.1)

$G\,\phi \equiv \neg F \neg \phi$

$F\,\phi \equiv \neg G \neg \phi$

Natural Language:

It's impossible to reach a state where the system is started but not ready.

Assumed atoms:

- started
- ready

$\neg F\,(\text{started} \wedge \neg \text{ready})$

LTL Formulation

$G\,(\neg(\text{started} \wedge \neg \text{ready}))$

$G\,(\neg \text{started} \vee \text{ready}) \rightarrow G\,(\text{started} \Rightarrow \text{ready})$

Formulating Natural Language in LTL (2.2)

Natural Language:

Whenever a request is made,
it will be acknowledged eventually.

no starvation

Assumed atoms:

- requested
- acknowledged

LTL Formulation

$$G(\text{requested} \Rightarrow F\,\text{ack.})$$

Formulating Natural Language in LTL (2.3)

Natural Language:

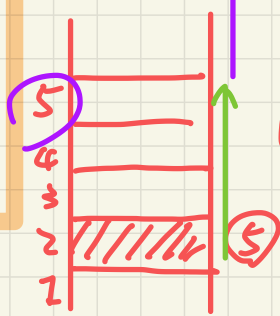
An elevator traveling upwards at the 2nd floor
does not change its direction
when it has passengers wishing to to to the 5th floor.

Assumed atoms:

- floor2, floor5
- directionUp
- buttonPressed5

elevator state

LTL Formulation



$$G\big(\text{floor2} \wedge \text{buttonPressed5} \Rightarrow (\text{directionUp}\ U\ \text{floor5})\big)$$

W X